2019 Lec 10

2019年5月23日 13:49

LP-rounding for set cover.

LP OPT $\{p_1, \dots, p_m\}$

Algo: 以 p_i 概率 choose set S_i $c(S_i)$

element u: $\sum_{S: u \in S} p_S \geq 1$

$p_{i_1} + p_{i_2} + \dots + p_{i_k} \geq 1$

$\Pr(u \text{ is covered}) = 1 - \prod_{j=1}^{k}(1-p_{i_j}) \geq \dots$

$\geq 1 - \left(1-\frac{1}{k}\right)^k \geq 1 - \frac{1}{e}$

$\left(1-\frac{1}{e}\right) \cdot n$

Repeat t times.

$\Pr(u \text{ is covered}) \geq 1 - \left(\frac{1}{e}\right)^t$

$\Pr(\forall u, u \text{ is covered}) = 1 - \Pr(\exists u, u \text{ is not covered})$

set cover.

$\geq 1 - n \cdot \left(\frac{1}{e}\right)^t$

$t \approx 2\log n$

$\geq 1 - o(1)$

$E(c(\text{chosen sets})) = E(\text{Algo})$

$\leq t \times \sum_i p_i c(S_i) = t \cdot OPT \approx 2\log n \cdot OPT$

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set cover.

$\sum c(S) x_S$

s.t. $\forall e$. $\sum x_S \geq 1$ 至多 f 项

$S: e \in S$

$x_S \ge 0$.

LP-rounding f-ratio. $x_S \ge \frac{1}{f} \to 1$

$x_S < \frac{1}{f} \to 0$.

Primal LP

min $\sum c(S) x_S$

s.t. $\sum_{S: e \in S} x_S \ge 1$ ($\forall e$)

$x_S \ge 0$

Dual LP

max. $\sum y_e$.

s.t. $\sum_{e: e \in S} y_e \le c(S)$

$y_e \ge 0$.

① $x_S = 0$ OR $\sum_{e: e \in S} y_e = c(S)$ $\forall S$.

② $y_e = 0$ OR $1 \le \sum_{S: e \in S} x_S \le f$. $\forall e$.

trivial.

$\alpha = 1$ $\beta = f$.

1°. initial $x_S = 0$. $y_e = 0$.

① $x_S \ne 0 \Rightarrow \sum_{e: e \in S} y_e = c(S)$.

2°. Repeat.

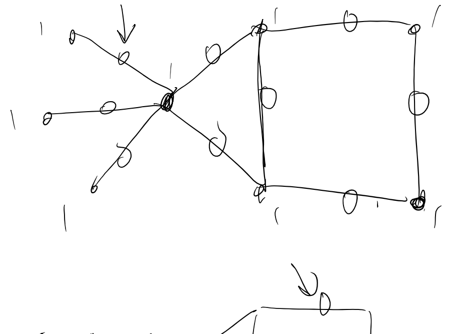
choose uncovered element e' $y_{e'} \uparrow$.

until $\exists S: e' \in S$ 且 $\sum_{e: e \in S} y_e = c(S)$ 成立

$x_S: 0 \to 1$.

initial

y_e:

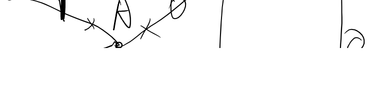


element: edge.

set: vertex.

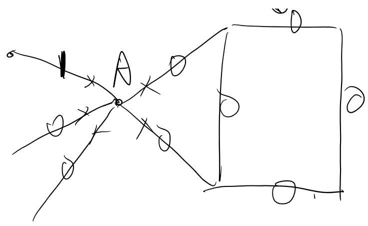
一个点都没选.

y_e:

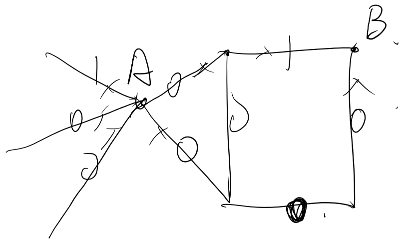


chosen vertex: {A}

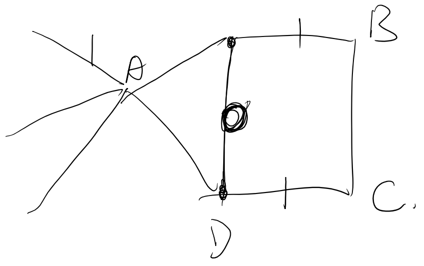
Ye:



chosen vertex : {A}



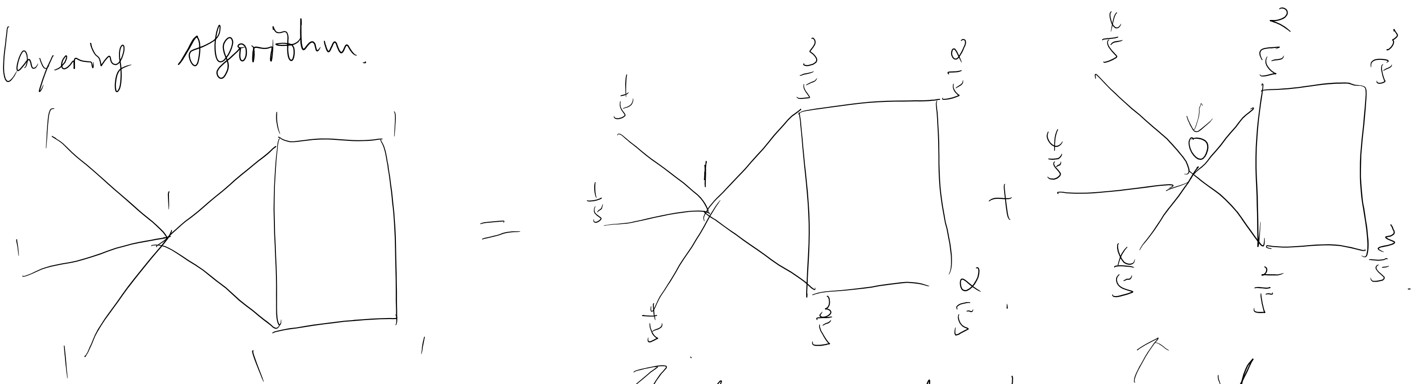
chosen vertex : {A, B}



{A, B, C}

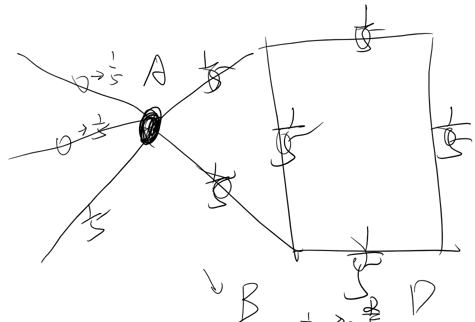
{A, B, C, D}

layering Algorithm

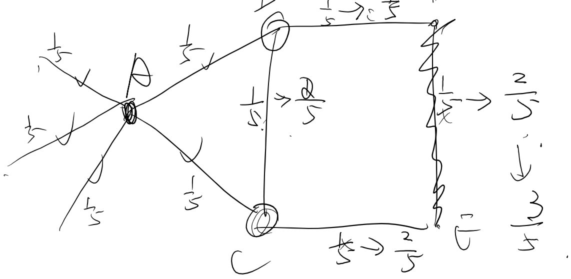


degree weighted graph

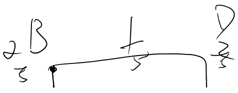
residue graph

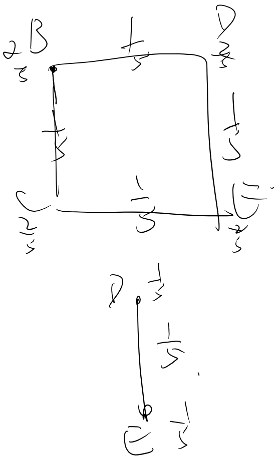


choose A.

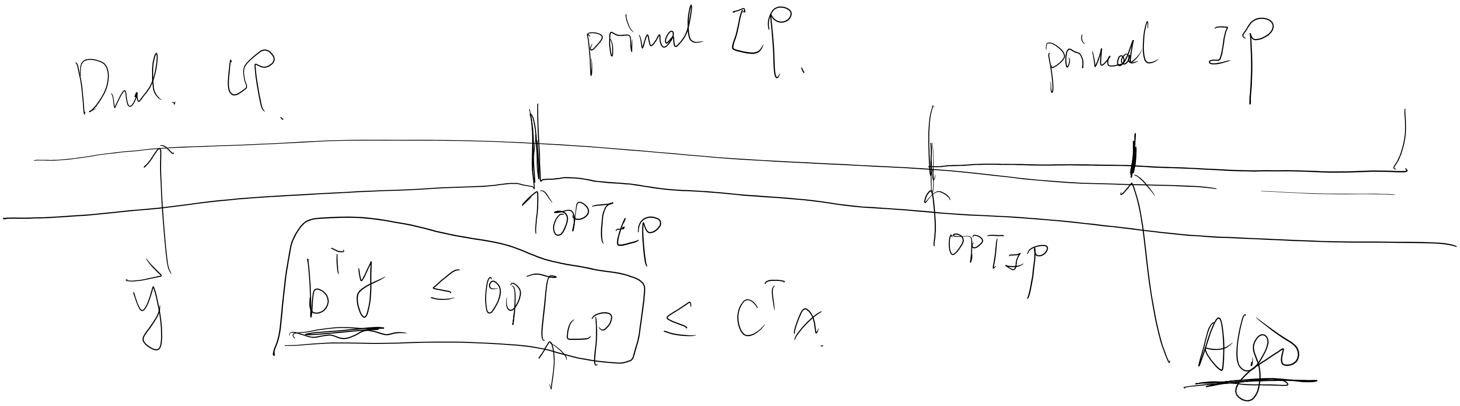


choose {A, B, C}





Primal. Dual   primal min   dual: max



$$b^T y \le OPT_{LP} \le c^T x$$

$$Algo. \le \alpha \cdot OPT$$

MAX - CUT.

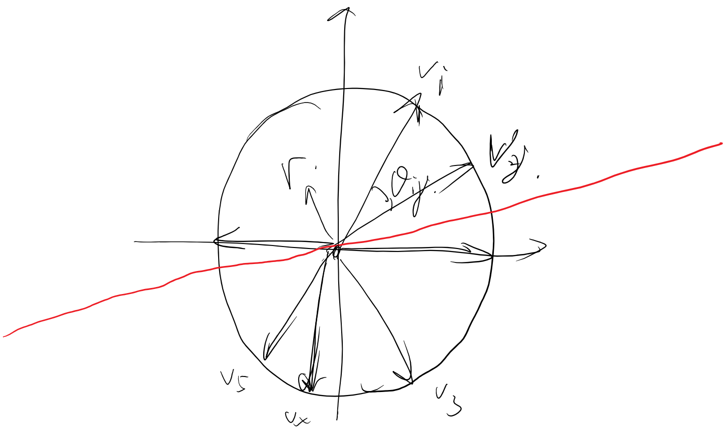
$$\max \ \frac{1}{2} \sum w_{ij} (1 - \vec{v_i} \cdot \vec{v_j}).$$

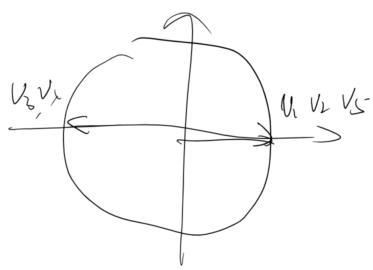
$$\text{s.t.} \quad \vec{v_i} \cdot \vec{v_i} = 1$$

$$\vec{v_1} \ \cdots \ \vec{v_n}$$

rounding

$$y_1 \ - \ y_n.$$





randomized algorithm.

random choose $\vec{r}$.

$\underline{S} = \{ v_i \mid v_i \cdot r \geq 0 \} \longrightarrow y_i = 1$

$\overline{S} = \{ v_i \mid v_i \cdot r < 0 \} \longrightarrow y_i = -1$

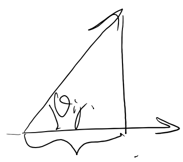
$E(Algo)$

$= E\left(\sum w_{ij} \dfrac{(1 - y_i y_j)}{2} \right)$

SDP OPT:

$\frac{1}{2} \sum w_{ij} (1 - \vec{v_i} \cdot \vec{v_j})$

$= \sum w_{ij} \dfrac{1 - \cos\theta_{ij}}{2}$



$= \sum w_{ij} \cdot \dfrac{\theta_{ij}}{\pi} \geq \underline{\alpha} \cdot OPT$

$$\min_{\theta_{ij}} \frac{\frac{\theta_{ij}}{\pi}}{\frac{1 - \cos\theta_{ij}}{2}} \approx 0.87856$$

$E(Algo) \geq 0.87856 \cdot OPT$

A: max. B. NPC.

$I_B \longrightarrow I_A \xrightarrow{\text{近似算法}} Algo$

1°. $Algo \geq \alpha \cdot m \Rightarrow OPT \geq \alpha \cdot m \Rightarrow$ B is satisfiable

2°. $\underline{Algo < \alpha \cdot m} \Rightarrow OPT < m \Rightarrow$ B is not satisfiable

if Algo is α-approximation $\Rightarrow$ $\exists$ poly-time algo for B.

$\Rightarrow P = NP$

$\Rightarrow P = NP$

if $P \neq NP$ $\Rightarrow$ there is no α-approx. algo for A.

B: